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Transient heat transfer in packed beds: The significance of the history term

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Abstract

Describing transient heat transfer in heterogeneous media is still a problem. This study was conducted to investigate if the consideration of the history term is a way to improve the description of this phenomenon. To that end the history term is analytically derived for a particle in a packed bed with stagnant fluid and its order of magnitude is estimated. The history term is significant in liquid–solid packed beds, whereas in the case of gas–solid packed beds the history term can usually be neglected except for very large heat fluxes as for instance in laser-pulse heating.

Keywords: Heat transfer; Transient; Packed bed; History term

1. Introduction

In several previous studies the transient heat transfer from (or to) a rigid sphere situated in a fluid of infinite extension has been examined [1–5]. It was shown by different derivations [2,4,5] that a history term has to be considered in the transient energy balance of the particle. The cases in which the history term may be of significance are somewhat controversial: While in [3] and [5] it was concluded that for $\beta \equiv (\tilde{c}_{p,f}/\tilde{c}_{p,s}) \rightarrow 0$ – as for instance in gas–solid flows – the history term is negligible, it has been shown recently in a numerical study [1], that the history term is also significant for $\beta > 0.01$. All abovementioned studies dealing with the history term in the energy balance of a particle have in common that the findings therein can only be attributed to dilute suspensions of particles in a stagnant fluid. So far no study has been conducted examining the significance of the history term in case of transient heat transfer in packed beds with stagnant flow.

The latter phenomenon is of special interest because it was observed in various experiments [6–10] that it can sometimes not be described by parabolic heat conduction (Eq. (1))

$$\vec{q}(\vec{r},t) = -\lambda_{\text{eff}} \nabla T(\vec{r},t). \tag{1}$$

A prerequisite for the validity of the parabolic equation is local thermal equilibrium, i.e. the temperatures of the fluid and solid phase are equal in every differential volume element. To account for local thermal non-equilibrium in transient heat transfer, the dual phase lag (DPL) model was proposed [11]. It was suggested that Eq. (1) needs to be modified as follows:

$$\vec{q}(\vec{r},t+\tau_{\text{q}}) = -\lambda_{\text{eff}} \nabla T(\vec{r},t+\tau_{\text{T}}). \tag{2}$$

τ_{q} and τ_{T} are referred to as lag times for heat flux and temperature gradient, respectively. Expanding $\vec{q}$ and ∇T in Taylor series, where the terms of higher than first order are neglected, yields:

$$\vec{q}(\vec{r},t) + \tau_{\text{q}} \frac{\partial \vec{q}(\vec{r},t)}{\partial t} = -\lambda_{\text{eff}} \left(\nabla T(\vec{r},t) + \tau_{\text{T}} \frac{\partial \nabla T(\vec{r},t)}{\partial t} \right). \tag{3}$$

Experimental studies [6,8] demonstrated that the heat transfer equation according to the DPL model (Eq. (3)) is better suited for transient heat transfer in heterogeneous media than Fourier's law (Eq. (1)). But the evalua-

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Nomenclature

Latin symbols

a	thermal diffusivity
A	area
c_p	mass specific heat capacity at constant pressure
$\tilde{c}_p$	volume specific heat capacity at constant pressure
d	diameter
h	roughness
L	distance between two parallel planes
N	number of particle layers
$\vec{q}$	heat flux vector
r	radial coordinate, radius
$\vec{r}$	position vector
t	time
T	temperature
u	generic quantity
V	volume
y	generic quantity

Greek symbols

β	ratio of volumetric heat capacities (fluid/solid)
γ	geometric parameter, ratio L/d_s
λ	thermal conductivity
Λ	mean free path
τ	(lag) time
θ	angle
ϱ	density

Subscripts

0	initial value
eff	effective value
c	pertains to inter-particle heat transfer by contact conduction
f	pertains to fluid
i	pertains to the inner zone of inter-particle heat transfer
j	pertains to the jth neighbour particle
max	maximum value
o	pertains to the outer zone
q	pertains to heat flux
r	pertains to radiation
s	pertains to particle/solid phase
T	pertains to temperature gradient
Δs	pertains to temperature step change at the particle's surface
Δj	pertains to temperature step change at the surface of the jth neighbour particle
∞	at large distance from particle

Superscript

*	dimensionless quantity

tion of the lag times is still a matter of discussion. In a very recent study [12] it was theoretically predicted that $\tau_T > \tau_q$. According to Eq. (2) this means that the heat flux depends on a temperature gradient of the future. This raises the question if the principle of causality is violated if $\tau_T > \tau_q$ holds. Fitting of experimental data resulted in the opposite conclusion [8,13]. Furthermore it has been found by fitting of experimental data [6] and by theory [12] that the lag times can be of the order of several 100 s. These large values indicate that the neglection of the higher order terms of the Taylor series expansion that was used in the derivation of Eq. (3) is not justified, meaning that the DPL model is not adequate. Hence, it is to be expected that at large values of the lag times τ_q or τ_T Eq. (3) does not hold.

In many studies $\tau_T = 0$ is adopted, resulting in the hyperbolic heat transfer equation:

$$\vec{q}(\vec{r},t) + \tau_q \frac{\partial \vec{q}(\vec{r},t)}{\partial t} = -\lambda_{\mathrm{eff}} \nabla T(\vec{r},t). \quad (4)$$

Eq. (4) is often referred to as Cattaneo–Vernotte [14,15] equation. It has also been successfully applied to various transient heat transfer processes in heterogeneous media [7,9,10]. But the experimental data to which the Cattaneo–Vernotte equation were applied, have been questioned seriously [16,17]. The order of magnitude of the lag time τ_q (also often referred to as relaxation time) is also a matter of controversy, as has been pointed out by Roetzel et al. [7].

In light of these considerations it can be concluded that the parabolic equation (Eq. (1)) can not be applied universally for transient heat transfer in heterogeneous media, but the establishment of a model that reliably predicts transient heat transfer remains a task for the scientific community.

One major shortcoming of the parabolic heat transfer equation (Eq. (1)) is that the effective thermal conductivity λ_{eff} only applies to steady-state heat transfer in heterogeneous media [18]. It will be shown in this study that the history term is a means to correct this error. The main objective of this study is, therefore, to investigate if the history term has a significant impact. If that is the case, then the fact that the parabolic equation (Eq. (1)) fails to describe transient heat transfer in heterogeneous media may be due to the neglection of the history term.

The paper is organized as follows: In Section 2 the history term for a rigid particle in a packed bed of spheres will be derived. A physical interpretation of the history term will be given. In Section 3 it will be estimated if the history term has a significant impact on transient heat transfer in heterogeneous media. After a discussion of the results (Section 4) a conclusion will be presented (Section 5).

2. Derivation of the history term for a particle in a packed bed

So far the history term was only derived for a dilute suspension of particles [2,4,5]. In a monodisperse packed bed, however, a spherical particle is in contact with an average of six neighbour particles [19]. The fluid in the interstitial region is assumed to be stagnant. It is furthermore assumed that the temperature in the solid particles is homogeneous (but not constant in time). This assumption is justified by the fact that the heat conductivities of solids are several orders of magnitude larger than those of gases. For metals this assumption also holds in combination with liquids.

There are in principle three different ways of heat transfer between the surfaces of two contacting particles in a stagnant fluid: radiation ($\dot{q}_\mathrm{r}$), conduction through the fluid, and conduction through the contact area ($\dot{q}_\mathrm{c}$). Slavin et al. [20] pointed out that the conduction through the fluid can be further categorized: In a region where the gap between the two spheres is less than two third of the molecular mean free path (Λ) it is more likely for a molecule to collide with the surface than with another molecule. The heat transfer within this inner region is referred to as $\dot{q}_\mathrm{i}$. In an outer region, where the gap is $>(2/3)\Lambda$, heat transfer occurs by "usual" conduction ($\dot{q}_\mathrm{o}$).

Thus, the energy balance of a particle with homogeneous temperature in a packed bed without heat source or sink reads:

$$\varrho_s c_{p,s} V_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = -\sum_{j=1}^{6}\left(\int_{A_{\mathrm{c},j}} \vec{\dot{q}}_{\mathrm{c},j}\,\mathrm{d}\vec{A} + \int_{A_{\mathrm{i},j}} \vec{\dot{q}}_{\mathrm{i},j}\,\mathrm{d}\vec{A} + \int_{A_{\mathrm{o},j}} \vec{\dot{q}}_{\mathrm{o},j}\,\mathrm{d}\vec{A} + \int_{A_{\mathrm{r},j}} \vec{\dot{q}}_{\mathrm{r},j}\,\mathrm{d}\vec{A}\right). \tag{5}$$

Here j is the index for one of the six neighbour particles. $A_{\mathrm{c},j}$, $A_{\mathrm{i},j}$, $A_{\mathrm{o},j}$, and $A_{\mathrm{r},j}$ are the areas of the particle surface through which $\dot{q}_{\mathrm{c},j}$, $\dot{q}_{\mathrm{i},j}$, $\dot{q}_{\mathrm{o},j}$, and $\dot{q}_{\mathrm{r},j}$ are transferred, respectively. The boundary of the inner contact region $A_{\mathrm{i},j}$ is given by the angle θ_i. According to Slavin et al. [20]:

$$\begin{aligned} \theta_\mathrm{i} &= \tfrac{(2/3)\Lambda - 2h}{r_s^2} && \text{if } \tfrac{2}{3}\Lambda > 2h,\\ \theta_\mathrm{i} &= 0 && \text{if } \tfrac{2}{3}\Lambda \leqslant 2h. \end{aligned} \tag{6}$$

Here, h is the roughness of the particle surface. Angle θ_o, the outer boundary for $A_{\mathrm{o},j}$, can only be roughly estimated to be 60°.

In a first study Slavin et al. [21] concluded that the conductance across the particle–particle contact was negligible except under conditions of low gas pressures. In a second study [20] they also showed that at gas pressures above 0.048 bar and at temperatures between 300 and 900 K the heat conduction in the outer fluid region $\dot{q}_\mathrm{o}$ dominates all other modes of heat transfer between contacting spheres. Thus, if no heat sink or source is present, then the energy balance of the particle reduces to:

$$\varrho_s c_{p,s} V_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = -\sum_{j=1}^{6}\int_{A_{\mathrm{o},j}} \vec{\dot{q}}_{\mathrm{o},j}\,\mathrm{d}\vec{A} = \sum_{j=1}^{6}\int_{A_{\mathrm{o},j}} (\lambda_{f,j}\nabla T_{f,j})\,\mathrm{d}A. \tag{7}$$

That is: the energy of the particle is altered only by heat conduction through the fluid (under the conditions stated above). In general $\dot{q}_{\mathrm{o},j}$ is not distributed homogeneously over $A_{\mathrm{o},j}$. Therefore the integrals in Eq. (7) cannot be determined analytically. The latter can be made possible if the geometry of the contact region is modeled as shown in Fig. 1. According to this model the heat transfer between two contacting spheres is represented by one-dimensional heat conduction between two parallel planes. Under the assumption that the fluid temperature between these two parallel planes is only dependent on time and coordinate x (see Fig. 2) the energy balance of the particle can be simplified to:

$$\varrho_s c_{p,s} V_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = \sum_{j=1}^{6}\lambda_{f,j}\left(\frac{\partial T_{f,j}(0,t)}{\partial x}\right)A_{\mathrm{o},j}. \tag{8}$$

The gradient $(\partial T_{f,j}/\partial x)$ can now be determined analytically which will be outlined in the following.

If the dimensionless responses $y_k^*(t)$ of an output variable $y(t)$ to step changes of various input variables $u_k(t)$ are known, then Duhamel's theorem reads [22]:

$$y(t) = \sum_{k=1}^{n}\left(u_k(t=0)y_k^*(t) + \int_0^t y_k^*(t-\tau)\frac{\mathrm{d}u_k(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau\right). \tag{9}$$

This superposition of several input variables $u_k(t)$ is only possible if the dimensionless responses $y_k^*(t)$ do not depend on the values of $u_{l\neq k}(t)$. It will be demonstrated in the course of this study, that this condition is valid in our case.

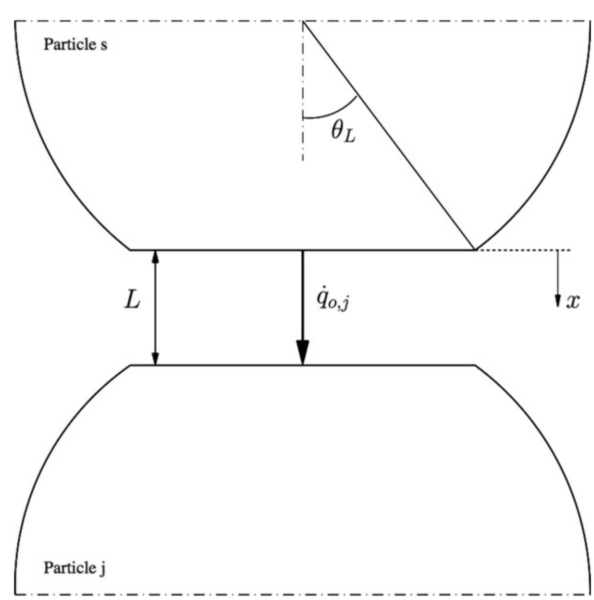

Fig. 1. Model for the zone between two contacting spheres.

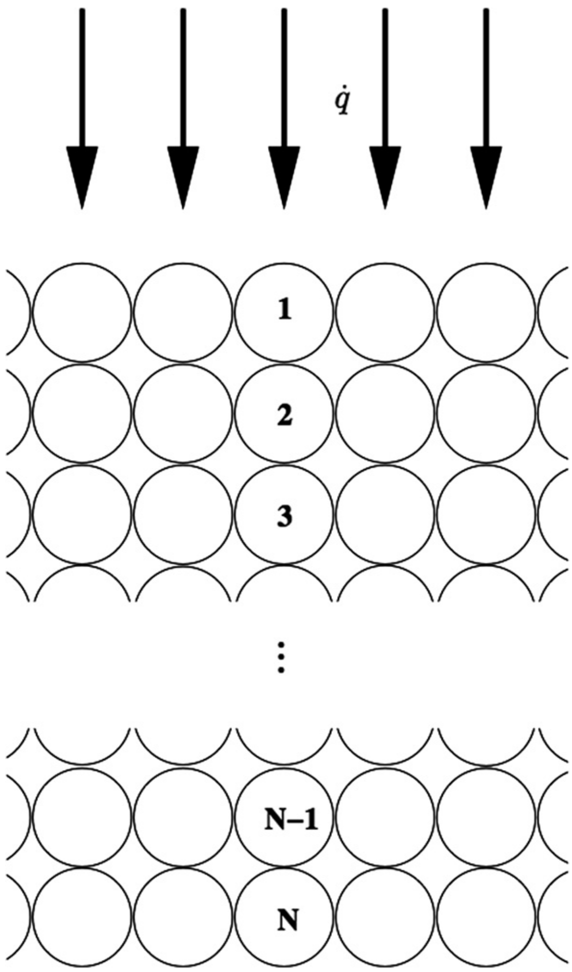

Fig. 2. Setup of the thought experiment.

In our case it is advantageous to choose $T_{f,j}(x,t) - T_0$ as the output quantity $y(t)$. $T_{f,j}(x,t)$ is the fluid temperature between particle s and particle j, while T_0 is the homogeneous initial temperature of the whole packed bed. The input quantities can be identified with the temperatures of the two parallel planes, i.e. the temperatures of two neighbour particles, $T_s(t)$ and $T_j(t)$, respectively. It is, again, useful to relate these temperatures to the initial temperature T_0. Thus, Duhamel's theorem applied to our problem reads:

$$T_{f,j}(x,t) - T_0 = \int_0^t T^*_{\Delta s}(x, t-\tau)\frac{dT_s(\tau)}{d\tau}d\tau + \int_0^t T^*_{\Delta j}(x, t-\tau)\frac{dT_j(\tau)}{d\tau}d\tau. \quad (10)$$

$T^*_{\Delta s}$ and $T^*_{\Delta j}$ are the dimensionless transient responses of the fluid temperature field to step changes of the temperatures T_s and T_j, respectively. These responses will be analytically derived in the next paragraphs. Eq. (10) can be further simplified by taking the derivative with respect to x, since we are actually interested in the gradient at the surface of the particle (i.e. at $x = 0$):

$$\frac{\partial T_{f,j}(0,t)}{\partial x} = \int_0^t \left[\left(\frac{\partial T^*_{\Delta s}(0,t-\tau)}{\partial x}\right)\frac{dT_s(\tau)}{d\tau} + \left(\frac{\partial T^*_{\Delta j}(0,t-\tau)}{\partial x}\right)\frac{dT_j(\tau)}{d\tau}\right]d\tau. \quad (11)$$

Eq. (11) shows that we have to determine the time-dependent gradients of the dimensionless fluid temperature at $x = 0$ for step changes of T_s and T_j, respectively. This will be done in the following.

The analytical solution for the one-dimensional transient temperature field $T_{f,j}(x,t)$ in a stagnant fluid bounded by two parallel planes is given in [23]. If the temperatures at the two boundaries are kept fixed at $T_{f,j}(0, t > 0) = T_s$ and $T_{f,j}(L, t > 0) = T_j$, respectively, then:

$$T_{f,j}(x,t) = T_s + (T_j - T_s)\frac{x}{L} + \cdots + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n T_j - T_s}{n}\sin\frac{n\pi x}{L}\exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right] + \frac{2}{L}\sum_{n=1}^{\infty}\sin\frac{n\pi x}{L}\exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right] \times \int_0^L T(x',0)\sin\frac{n\pi x'}{L}dx'. \quad (12)$$

For a homogeneous initial temperature field, i.e., $T_{f,j}(x,0) = T_0$, the integral on the right-hand side can be evaluated as follows:

$$\int_0^L T_0 \sin\frac{n\pi x'}{L}dx' = -\frac{LT_0}{n\pi}\left[\cos\frac{n\pi x}{L}\right]_0^L = -\frac{LT_0}{n\pi}[(-1)^n - 1]. \quad (13)$$

Hence, the transient temperature field can be rewritten as

$$T_{f,j}(x,t) = T_s + (T_j - T_s)\frac{x}{L} + \ldots + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n(T_j - T_0) - (T_s - T_0)}{n}\sin\frac{n\pi x}{L} \times \exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right]. \quad (14)$$

A temperature step change at $x = L$ is equivalent to setting $T_s(t > 0) = T_0$. The influence of a change of T_s on the fluid temperature gradient at $x = 0$ will be evaluated seperately (see Eq. (18)). The resulting transient temperature field is therefore:

$$T_{\Delta j}(x,t) = T_0 + (T_j - T_0)\left(\frac{x}{L} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{L} \times \exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right]\right). \quad (15)$$

The derivative with respect to x is

$$\frac{\partial T_{\Delta j}(x,t)}{\partial x} = \frac{T_j - T_0}{L}\left(1 + 2\sum_{n=1}^{\infty}(-1)^n\cos\frac{n\pi x}{L}\exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right]\right). \quad (16)$$

For the gradient of the dimensionless temperature at $x = 0$ we get:

$$\frac{\partial T^*_{\Delta j}(0,t)}{\partial x} = \frac{1}{T_j - T_0}\frac{\partial T_{\Delta j}(0,t)}{\partial x} = \frac{1}{L}\left(1 + 2\sum_{n=1}^{\infty}(-1)^n \exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right]\right). \tag{17}$$

Hence, the second gradient in the integral of Eq. (11) is known. The first gradient in that equation can be determined analogously, yielding:

$$\frac{\partial T^*_{\Delta s}(0,t)}{\partial x} = \frac{1}{T_s - T_0}\frac{\partial T_{\Delta s}(0,t)}{\partial x} = -\frac{1}{L}\left(1 + 2\sum_{n=1}^{\infty} \exp\left[-\frac{a_{f,j}n^2\pi^2 t}{L^2}\right]\right). \tag{18}$$

The analytical solutions for the two gradients (Eqs. (17) and (18)) show that these responses do not depend on either one of the two input parameters T_j and T_s. Therefore Duhamel's theorem – as written in Eqs. (10) and (11) – can be applied in our case. Substituting the gradients in the integral of Eq. (11) with Eqs. (17) and (18) yields:

$$\begin{aligned}\frac{\partial T_{f,j}(0,t)}{\partial x} &= -\frac{1}{L}\int_0^t\left(1+2\sum_{n=1}^{\infty}\exp\left[-\frac{a_{f,j}n^2\pi^2(t-\tau)}{L^2}\right]\right)\frac{dT_s}{d\tau}d\tau \\ &\quad + \frac{1}{L}\int_0^t\left(1+2\sum_{n=1}^{\infty}(-1)^n\exp\left[-\frac{a_{f,j}n^2\pi^2(t-\tau)}{L^2}\right]\right) \\ &\quad \times\frac{dT_j}{d\tau}d\tau \\ \Rightarrow \frac{\partial T_{f,j}(0,t)}{\partial x} &= \frac{T_j - T_s}{L} + \cdots \\ &\quad + \frac{2}{L}\int_0^t\sum_{n=1}^{\infty}\exp\left[-\frac{a_{f,j}n^2\pi^2(t-\tau)}{L^2}\right] \\ &\quad \times\left((-1)^n\frac{dT_j}{d\tau} - \frac{dT_s}{d\tau}\right)d\tau.\end{aligned} \tag{19}$$

This result can now be applied to the energy balance of the particle, Eq. (8), resulting in

$$\begin{aligned}\varrho_s c_{p,s} V_s \frac{dT_s}{dt} &= \sum_{j=1}^{6} A_{o,j}\lambda_{f,j}\frac{T_j - T_s}{L} + \ldots \\ &\quad + \sum_{j=1}^{6} A_{o,j}\frac{2\lambda_{f,j}}{L}\int_0^t\sum_{n=1}^{\infty}\exp\left[-\frac{a_{f,j}n^2\pi^2(t-\tau)}{L^2}\right] \\ &\quad \times\left((-1)^n\frac{dT_j}{d\tau} - \frac{dT_s}{d\tau}\right)d\tau.\end{aligned} \tag{20}$$

For the sake of brevity the second term on the right-hand side of Eq. (20) will be referred to as f_{HT} in the following.

For a physical interpretation of the history term let us at first focus on the first term on the right-hand side of Eq. (20). This term describes the heat flow through $A_{o,j}$ if the fluid temperature fields are at steady-state. The ratio $\lambda_{f,j}/L$ is equal to the heat transfer coefficient. If this relation for steady-state heat transfer between two parallel planes is applied to transient heat transfer as well, then an error will be introduced. This error is corrected by means of the history term. Therefore, f_{HT} can be regarded as a correction term.

3. Significance of the history term for a particle in a packed bed

In order to examine the significance of the history term of a particle in a packed bed let us consider a thought experiment with a setup as depicted in Fig. 2. A cluster containing N particle layers and with homogeneous initial temperature T_0 is subjected to a constant heat flux $\dot{q}$. The heat transfer occurs one-dimensionally, i.e. perpendicular to the cluster surface. In this case only two of the six neighbours of a particle in the bulk are relevant. The temperature of the N-th particle layer at the bottom is held constant at T_0. The properties (ϱ, c_p, λ) of both fluid and solid are assumed to be constant. As stated above, the temperature of each particle is homogeneous, which is equivalent to the assumption that $\lambda_s \to \infty$. Under these conditions and by use of dimensionless variables defined by

$$t^* \equiv \frac{a_f t}{d_s^2} \quad \text{and} \quad T^* \equiv \frac{(T_{1,\infty} - T)}{(T_{1,\infty} - T_0)} \tag{21}$$

the energy balance of a particle i (Eq. 20) having two neighbours can be transformed into

$$\begin{aligned}\frac{2}{3\beta(2-\gamma)}\frac{dT^*_i}{dt^*} &= (T^*_{i-1} - T^*_i) + (T^*_{i+1} - T^*_i) + \cdots \\ &\quad + 2\int_0^{t^*}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{n\pi}{\gamma}\right)^2(t^* - \tau^*)\right] \\ &\quad \times\left((-1)^n\frac{dT^*_{i-1}}{d\tau^*} - \frac{dT^*_i}{d\tau^*}\right)d\tau^* \\ &\quad + 2\int_0^{t^*}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{n\pi}{\gamma}\right)^2(t^* - \tau^*)\right] \\ &\quad \times\left((-1)^n\frac{dT^*_{i+1}}{d\tau^*} - \frac{dT^*_i}{d\tau^*}\right)d\tau^*,\end{aligned} \tag{22}$$

with $V_s = \pi d_s^3/6$, $\gamma \equiv L/d_s$, and thus $A_o = \gamma(2-\gamma)\pi d_s^2/4$. The temperature of the particles at the bottom is held constant, i.e.

$$\frac{dT^*_N}{dt^*} = 0. \tag{23}$$

The dimensionless energy balance of a particle in the topmost layer reads:

$$\begin{aligned}\frac{2}{3\beta(2-\gamma)}\frac{dT^*_1}{dt^*} &= (T^*_2 - T^*_1) + \frac{1}{N-1} + \cdots \\ &\quad + 2\int_0^{t^*}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{n\pi}{\gamma}\right)^2(t^* - \tau^*)\right] \\ &\quad \times\left((-1)^n\frac{dT^*_2}{d\tau^*} - \frac{dT^*_1}{d\tau^*}\right)d\tau^*.\end{aligned} \tag{24}$$

For the derivation of the second term on the right-hand side of Eq. (24) the reader is referred to Appendix A.

The system of N differential equations (Eqs. (22)–(24)) can now be solved numerically. For this purpose the integrals are approximated by the trapez rule. The LIMEX solver [24] is used to integrate the system of differential equations.

It should be noted that the dimensionless transient temperature field in the cluster does neither depend on $\dot{q}$ nor d_s. Only the impacts of three parameters are to be investigated: γ and N are geometrical parameters of the model whereas the ratio of the volumetric heat capacities β is the only physical parameter and therefore of special interest.

In Fig. 3 a typical result of the numerical integration is shown. Depicted is the time-dependent dimensionless temperature of the particles in the topmost layer of the cluster (see Fig. 2). As is to be expected, the history term does not affect the steady-state temperature field. Neglection of the history term leads to an overestimation of the transient temperature. This is due to the fact that the transient heat transfer from particle to fluid is larger than the steady-state heat transfer. The difference in the dimensionless temperature that results from neglection of the history term (referred to as ΔT^* in the following) is also depicted in Fig. 3. The impact of the three aforementioned parameters on the dimensionless temperature field can be quantified by the maximum of ΔT^*.

In Fig. 4 the crucial quantity ΔT^* is plotted as a function of time for several particles (or particle layers) in the cluster. Clearly, ΔT^* of the topmost particle layer is larger than for all subsurface layers. This is due to the way the dimensionless temperature is defined. In the subsequent diagrams, where the impact of the three parameters will be investigated, the maximum value of this deviation for the topmost particle layer ($\Delta T^*_{\max}$) will be used as a means to quantify the significance of the history term.

The geometric parameter γ represents the ratio L/d_s. This quantity has to be estimated by considering at which distance L the real heat transfer geometry is represented

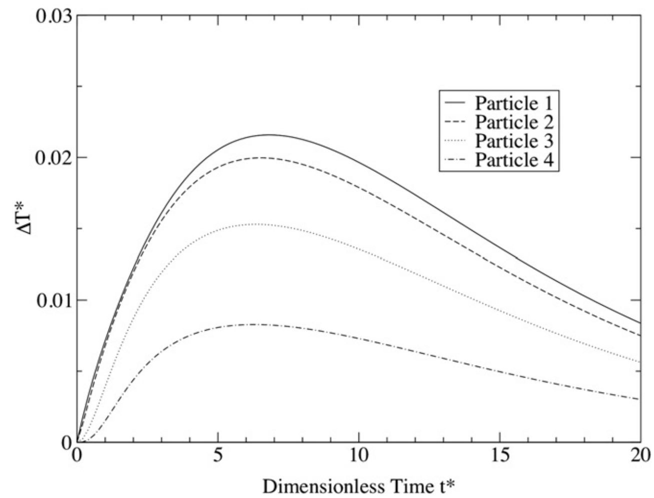

Fig. 4. Dimensionless error due to neglection of the history term for different particles (enumeration according to Fig. 2, $N = 5$, $\gamma = 0.2$, $\beta = 1.0$).

best by the parallel planes model (Fig. 1). There are two boundary cases: For $\gamma \to 0$ the temperature field between the almost touching parallel planes will reach steady-state quasi instantaneously, which is why the impact of the history term should vanish ($\Delta T^*_{\max} \to 0$). The largest geometrically meaningful value of γ is 1. In this case $\Delta T^*_{\max}$ should be the largest since it takes the longest time to reach the steady-state temperature field. The numerical results depicted in Fig. 5 are in agreement with these considerations. The graph also shows that for $0.1 < \gamma < 0.5$ the order of magnitude of $\Delta T^*_{\max}$ does not change. For examining the significance of the history term it is sufficient to estimate the order of magnitude of $\Delta T^*_{\max}$. Therefore γ will be set arbitrarily to 0.2 in the following.

The dependence of $\Delta T^*_{\max}$ on the number of particle layers N is illustrated in Fig. 6. For an understanding it is again useful to consider the two border cases: If $N = 1$ then

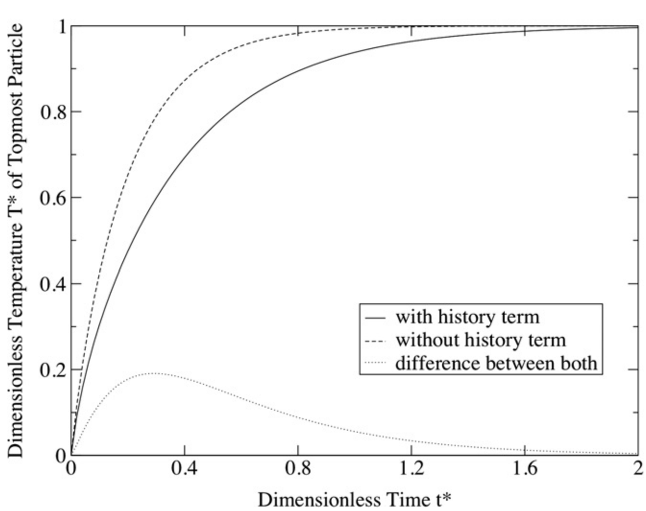

Fig. 3. Dimensionless temperature of the topmost particle layer with and without consideration of the history term ($N = 3$, $\gamma = 0.2$, $\beta = 1.0$).

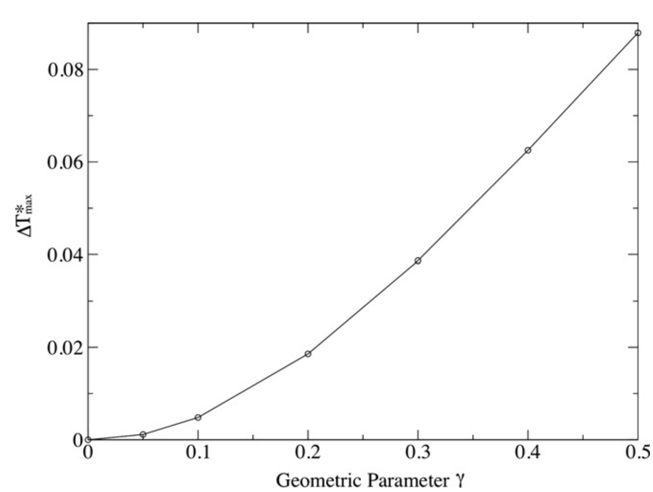

Fig. 5. Maximum dimensionless error due to neglection of the history term in dependence of the geometrical parameter γ ($N = 3$, $\beta = 1.0$).

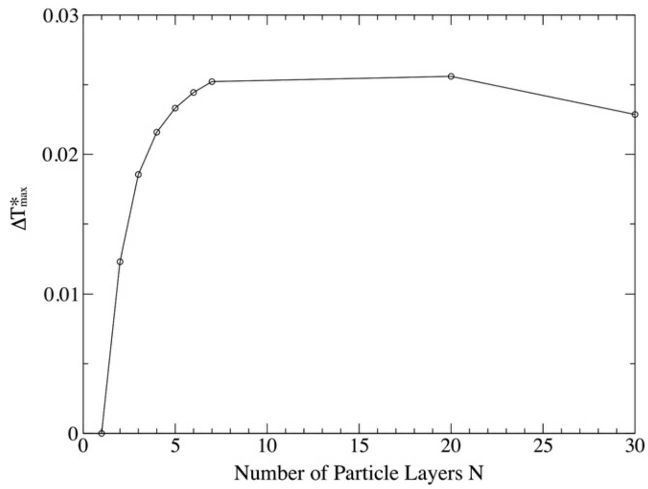

Fig. 6. Maximum dimensionless error due to neglection of the history term in dependence of the number of particle layers ($\gamma = 0.2$, $\beta = 1.0$).

the history term has no impact because the temperature of the particle layer is held constant. Thus $\Delta T^*_{max} = 0$. For $N \to \infty$ the denominator in the definition of the dimensionless temperature (Eq. (21)) is infinite. The temperature difference due to neglection of the history term is finite. Therefore, $\Delta T^*_{max} \to 0$. In light of these considerations it is clear why a vulcano-type behaviour as depicted in Fig. 6 can be observed. The main conclusion that can be drawn is that for $1 < N < 30$ the order of magnitude of ΔT^*_{max} is independent from N. Therefore N will be set arbitrarily to 3 in the following.

Having demonstrated that the geometrical parameters γ and N do not change the order of magnitude of ΔT^*_{max} it turns out that the physical parameter β is of fundamental importance as far as the significance of the history term is concerned. The major result of this study is depicted in Fig. 7. It can be concluded that the history term is significant for transient heat transfer in liquid–solid systems ($\beta \approx 10^0$). For gas–solid systems ($\beta \approx 10^{-3}$) the history term can be neglected.

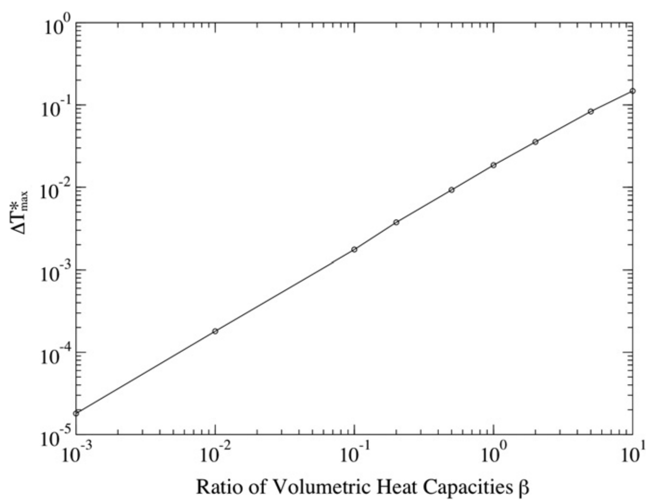

Fig. 7. Maximum dimensionless error due to neglection of the history term in dependence of the volumetric heat capacity ratio β ($N = 3$, $\gamma = 0.2$).

4. Discussion

The result depicted in Fig. 7 can be understood as follows: If the fluid in the interstitial region between two neighbouring solid particles has a significantly lower volumetric heat capacity compared to the solid (as is the case for gas–solid systems), then the temperature of the solid changes much slower than the temperature of the fluid. In other words: The temperature field in the fluid reaches steady-state quasi instantaneously. Therefore, the error that is introduced by using steady-state relationships for the heat transfer coefficient is very small. The history term (which corrects this error) can, thus, be neglected, except for very high heating rates which will be discussed below.

However, if the fluid has a volumetric heat capacity which is in the same order of magnitude as the one of the solid, then the time required for the temperature field in the fluid to reach steady-state is significantly long. Therefore, it is not justified to use steady-state relationships for the heat transfer coefficient (or effective heat conductivites) unless they are corrected by the history term.

It is to be expected that the larger the heat flux $\dot{q}$ the more significant the history term, since the change of temperature with time increases and, thus, the deviation from steady-state in the transient regime. This expectation may be regarded as contradictory to the fact that ΔT^*_{max} does not depend on $\dot{q}$. But the dependence is "hidden" in the definition of the dimensionless temperature (Eq. (21)), since in case of our thought experiment:

$$\dot{q} = \lambda_{eff} \frac{(T_{1,\infty} - T_0)}{(N-1)d_s}. \tag{25}$$

Therefore, it can be concluded that the absolute error (in K) due to neglection of the history term is proportional to the heat flux. This shall be illustrated in the following.

In a previous study [25] a packed bed of glass spheres in air ($\beta \approx 7.4 \times 10^{-4}$) with a height of 0.041 m was heated by a lamp ($\dot{q} = 880\ \mathrm{W/m^2}$). Fig. 7 shows that in this case $\Delta T^*_{max} \approx 10^{-5}$. With $\lambda_{eff} = 0.35\ \mathrm{W/(m\ K)}$, as determined in that study, it follows that $(T_{1,\infty} - T_0) \approx 100$ K. Therefore, neglection of the history term leads to an error of approximately 10^{-3} K, which is insignificant.

Jiang et al. [8] carried out experiments with laser pulse heating of $\dot{q} = 2.27 \times 10^9\ \mathrm{W/m^2}$. If this heat flux would have been imposed on the packed bed used by Polesek, then the error would be of the order of 10^3 K. Jiang et al. were able to show that the larger $\dot{q}$ the stronger the non-Fourier heat conduction behaviour. This experimental finding is in line with our argumentation.

5. Conclusion

In this study it has been shown that the history term can be interpreted as a correction term. It corrects the error of applying heat transfer coefficients (as well as effective thermal conductivites) to transient processes. These quantities are valid for steady-state processes only and must be corrected if they are applied to transient processes. The latter can be done by considering the history term.

For the first time a derivation of the history term based on a geometry of finite dimension has been presented resulting in a relation where no artificial quantities exist (as opposed to previous studies). Only measurable quantities are contained.

The history term can be neglected in most cases of transient heat transfer in gas–solid packed beds. It can not be neglected if the heat flux is very large ($\dot{q} > 10^6$ W/m^2), as is the case in laser pulse heatings. In liquid–solid packed beds the history term is always significant.

The fact that transient heat transfer in heterogeneous media can sometimes not be described by Fourier's law is in part, but not primarily due to the fact that the effective thermal conductivity is only applicable to steady-state processes.

Appendix A. Dimensionless energy balance of a particle in the topmost layer

For a particle in the topmost layer a term including the external heat flux $\dot{q}$ has to be added to the energy balance as written in Eq. (20). Substituting the temperature and the time by the dimensionless variables as defined in Section 3 leads to the following equation:

$$\frac{2}{3\beta(2-\gamma)}\frac{\mathrm{d}T_1^*}{\mathrm{d}t^*} = (T_2^* - T_1^*) + \frac{1}{(2-\gamma)}\frac{\dot{q}d_s}{\lambda_f(T_0 - T_{1,\infty})} + \cdots$$
$$+ 2\int_0^{t^*}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{n\pi}{\gamma}\right)^2(t^*-\tau^*)\right] \times \left((-1)^n\frac{\mathrm{d}T_2^*}{\mathrm{d}\tau^*} - \frac{\mathrm{d}T_1^*}{\mathrm{d}\tau^*}\right)\mathrm{d}\tau^*. \quad (A.1)$$

For $t \to \infty$ this balance reduces to

$$0 = (T_{2,\infty}^* - T_{1,\infty}^*) + \frac{1}{(2-\gamma)}\frac{\dot{q}d_s}{\lambda_f(T_0 - T_{1,\infty})}. \quad (A.2)$$

For the case of constant properties of solid and fluid phase the temperature field in the particle chain is linear at steady-state, which is why

$$\frac{T_{2,\infty}^* - T_{1,\infty}^*}{d_s} = \frac{T_{N,\infty}^* - T_{1,\infty}^*}{(N-1)d_s} = -\frac{1}{(N-1)d_s}. \quad (A.3)$$

With this relation and since $T_{N,\infty} = T_0$ Eq. (A.2) can be rewritten as

$$0 = -\frac{1}{(N-1)} + \frac{1}{(2-\gamma)}\frac{\dot{q}d_s}{\lambda_f(T_0 - T_{1,\infty})}. \quad (A.4)$$

Solving this equation for $\dot{q}$ yields

$$\dot{q} = \left(\frac{2-\gamma}{N-1}\right)\frac{\lambda_f(T_0 - T_{1,\infty})}{d_s}. \quad (A.5)$$

Substituting $\dot{q}$ in Eq. (A.2) with Eq. (A.5) yields Eq. (24).

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